crack, Trans. ASME, Ser. E., 3, 3, 1966.
2. GOL'DSHTEIN R.V., RYSKOV I.N. and SALGANIK R.L., A central transverse crack in an elastic strip, Izv. Akad. Nauk, Mekhan. Tverd. Tela, 4, 1969.
3. SNEDDON I.N. and SRIVASTAV R.P., The stress field in the vicinity of a Griffith crack in a strip of finite width, Intern. J. Engng. Sci., 9, 5, 1971.
4. ISIDA M., Methods of Laurent series expansion for internal crack problems. Methods of Analysis and Solutions of Crack Problems, Noordhoff, Leyden, 1973.
5. BENTHEM J.P. and KOITER W.T., Asymptotic approximations to crack problems. Methods of Analysis and Solutions of Crack Problems, Noordhoff, Leyden, 1973.
6. KEER L.M. and FREEDMAN J.M., Tensile strip with edge cracks, Intern. J. Engng Sci. 11, 12, 1973.
7. KRENK S., On the elastic strip with an internal crack, Intern. J. Solids and Struct., 11, 6, 1975.
8. PANASYUK V.V., SAVRUK M.P. and DATSYSHIN A.P., Stress Distribution Around Cracks in Plates and Shells. Naukova Dumka, Kiev, 1976.
9. CARDEW G.E. and HOWARD I.C., An edge-crack in an elastic strip and related problems in fracture mechanics and viscous flow, Intern. J. Engng. Sci., 14, 4, 1976.
10. MUSKHELISHVILI N.I., Certain Fundamental Problems of Mathematical Elasticity Theory, Nauka, Moscow, 1966.
11. KARPENKO L.N., On a method of computing the state of stress in the neighbourhood of a shallow drift traversed along a vertical coal stratum, Fiz-tekhn. Probl. Razrab. Polezn. Iskop., 4, 1965.
12. SZEGO G., Orthogonal Polynomials /Russian translation/, Fizmatgiz, Moscow, 1962.
13. KARPENKO L.M., On the mapping of functions by using Jacobi polynomials and the evaluation of some integrals of Cauchy type, Visn. Kiiv Univ., Ser. Matematiki ta mekhaniki, 13, 1971.
14. BOIKO A.V. and KARPENKO L.N., On some numerical methods for the solution of the plane elasticity problem for bodies with cracks by means of singular integral equations, Intern. J. Fracture, 17, 4, 1981.

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DIFFUSION CHARGING OF PARTICLES IN ONE-DIMENSIONAL WEAKLY IONIZED AEROSOL FLOWS*

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Electrohydrodynamics is used in /1, 2/ to study one-dimensional flows of aerosol particles carrying a bipolar charge in electric field, in the case when the parameters of the electrohydrodynamic (EHD) interaction between the phases are small. It is assumed that the radius of the aerosol particles is small and that the charging process is governed by the thermal motion of the ions towards their surface. The case of large Peclet numbers is considered, the numbers constructed in accordance with the characteristic dimension of the problem, i.e. by neglecting the contribution of diffusion towards the total macroscopic flows of the ions. The reaction rate at which the ions transfer their charge to the particles, is assumed to be finite. A digital computer is used to study the dependence of the flow parameters on the reaction rate constant and the particle density. The results of the calculations are compared with the analytic solution of the problem obtained for low-concentration aerosols in the case of large electrical Reynolds numbers.

EHD flows of weakly ionized aerosols with volume ion sources occur in various natural and technological processes caused, for example, by external radioactivity /1-3/. In such flows the particles of the disperse phase can become charged as a result of precipitation of ions of predominantly one sign. In order to study the special features of the interphase charge transfer in weakly ionized aerosols, in the presence of volume ionization, it is best to study

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their one-dimensional flows. This itself is of interest, since nearly one-dimensional flows are encountered in a number of EHD systems /4/.

Let us consider, in the half-space $x \geqslant 0$, a one-dimensional steady flow of a weakly ionized aerosol along the x axis of a Cartesian coordinate system. Let the electric field strength be also directed along the x axis and let the parameters of EHD interaction for the gas and disperse phase be small. Then we can assume that the velocities of the gas and aerosol particles are the same and equal to $u = \text{const} > 0$. The ion concentration distribution $n_\pm$, the charge of the particles e_p and the electric field strength E are described by the following system of equations:

$$\frac{d}{dx}[n_\pm(u \pm bE)] = \beta - \alpha n_+ n_- - n_p i_\pm \tag{1}$$

$$\frac{dE}{dx} = 4\pi(en_+ - en_- + e_p n_p), \quad u\frac{de_p}{dx} = e(i_+ - i_-)$$

$$i_\pm = \pm 4\pi n_\pm e_p b\left[\left(1 \pm \frac{e_p b}{a^2 K}\right)\exp\left(\pm\frac{e_p b}{aD}\right) - 1\right]^{-1}$$

Here b is the absolute value of the ionic mobility coefficient, which is assumed to be the same for ions of both signs and is connected with the coefficient of ionic diffusion D by the Einstein relation, β is the local rate of ionization of the gas, α is the ion recombination coefficient and e is the absolute value of the electric charge on the ion. The expression describing the ion flows towards the particle $i_\pm$ is written under the assumption that on reaching the surface of the particle the ions transfer their charge to it, and the surface reaction rate constants for the ions of both signs are qual to K /5/.

We shall assume that no ions are formed at the surface $x = 0$, that the particles have zero charge and that the electric field is known and equal to E°, i.e. that when $x = 0$, we have the initial conditions

$$I_\pm \equiv (u \pm bE)\,n_\pm = 0, \quad e_p = 0, \quad E = E^\circ, \quad 0 < E^\circ < u/b \tag{2}$$

Note that in writing Eqs.(1) and (2) we assumed that the contribution of diffusion towards the complete ion flows $I_\pm$ can be neglected. This requires that the condition $\text{Pe} = Lu/D \gg 1$ should hold, where L is the characteristic dimension of the problem. Since the problem in question has no characteristic geometrical dimension, it follows that the quality L represents the characteristic time needed for the state of equilibrium to be established (strictly speaking, this is attained in the limit as $x \to \infty$), due to the processes of recombination of the ions and their precipitation on the particles. Therefore $L \sim u \min(\tau_{ii}, \tau_\pm)$ where $\tau_{ii} = (\alpha n_s)^{-1}$, $\tau_\pm = (4\pi a D n_p)^{-1}$ are the characteristic times of change in ion concentration caused by their mutual recombination and precipitation on the particle, $n_s = \sqrt{\beta/\alpha}$ is the equilibrium concentration of the ions in the pure gas.

For example, for air under normal conditions we have $b \sim 2 \times 10^{-4}$ m^2/V.sec and $\alpha \sim 2 \times 10^{-12}$ m^3/sec. /3/. If a radioactive source is used for ionization with $\beta \sim 4 \times 10^{16}$ m^{-3}/sec /4/, then $\tau_{ii} \sim 4 \times 10^{-3}$ sec. When the particle radius is 10^{-6} m and their concentration is $n_p = 10^9$ m^{-3}, we have $\tau_\pm \sim 20$ sec. In this case $L \sim 4 \times 10^{-2}$ m corresponds to the stream velocity of 10 m/sec.

Let us introduce the following dimensionless quantities:

$$n_\pm^* = \frac{n_\pm}{n^\circ}, \quad n^\circ = \sqrt{\frac{\beta}{\alpha_L}}, \quad \alpha_L = 8\pi eb, \quad E^* = \frac{E}{E^\circ} \tag{3}$$

$$x^* = \frac{\sigma_L x}{u}, \quad \sigma_L = 8\pi ebn^\circ, \quad \gamma = \frac{\alpha}{\alpha_L}, \quad K^* = \frac{aK}{D}$$

$$e_p^* = \frac{e_p b}{aD}, \quad N = \frac{aDn_p}{2ebn^\circ} = \sqrt{\gamma}\,\frac{\tau_{ii}}{\tau_\pm}, \quad \text{Re}_E = \frac{u}{bE^\circ}$$

and henceforth omit the asterisks.

The parameter γ in gases under normal conditions falls within the range $(0{,}1;\,1)$ /6/ and $N \sim \tau_{ii}/\tau_\pm$. When the gases are sufficiently dense, the Langevin formula $\alpha = \alpha_L = 8\pi eb$ yields $\gamma = 1$, $N = \tau_{ii}/\tau_\pm$.

The system of Eqs.(1), (2) has no analytic solution and must be solved numerically for each set of defining parameters. The problem is, however, simplified when the parameters have their limiting values.

For example, let $\text{Re}_E \gg 1$, i.e. let the stream velocity be much greater than the ion migration rate under the action of the electric field. When $\text{Pe}_a = au/D \geqslant 1$, the inequality follows directly from the relation $\text{Re}_E = \text{Pe}_a D/(abE^\circ)$ and the assumed condition that a purely diffusive mechanism of charging the particle exists, which is true when $D/(abE^\circ) \gg 1$.

Applying the method of perturbations /7/ to problem (1), (2) written in dimensionless variables (3), we can represent all dependent variables in the form of the series

$$f = \sum_{n=0}^{\infty} f_n \left(\frac{1}{\mathrm{Re}_E}\right)^n$$

The following closed system of equations is obtained for the first non-zero terms:

$$\begin{aligned}
&\frac{dn_0}{dx} = 1 - \gamma n_0^2 - ANn_0, \quad \frac{dE_0}{dx} = -n_0E_0 \\
&\frac{de_{p1}}{dx} = -A\left[n_0E_0 + \left(N + \frac{1+K/2}{1+K}n_0\right)e_{p1}\right]; \quad A = \frac{K}{1+K} \\
&n_0(0) = e_{p1}(0) = 0, \quad E_0(0) = 1
\end{aligned} \tag{4}$$

The zeroth approximations in terms of the parameter $1/\mathrm{Re}_E$ of the dimensionless concentrations of the ions $n_\pm$ are the same, and are denoted by n_0.

When the parameter N is vanishingly small, an analytic expression for the dependence of e_{p1} on E_0 can be obtained from (4), as well as the corresponding relation $e_p(E)$, correct apart from terms of order $O(1/\mathrm{Re}_E)^2$

$$\begin{aligned}
&e_{p1} = \frac{AE_0}{\lambda}(1 - E_0^{-\lambda}), \quad e_p = \frac{AE_0}{\lambda\,\mathrm{Re}_E}(1 - E_0^{-\lambda}) \\
&2\lambda = 1 + (1+K)^{-2}
\end{aligned} \tag{5}$$

Figs.1-4 (the solid lines) show the results of a numerical solution of problem (1), (2) for $\gamma = 0{,}25$ which corresponds to dry air under normal conditions. Fig.1 shows the relations $e_p(E)$ for the case $N = 1$, $\mathrm{Re}_E = 1$ for various value of K. The relations $e_p(E)$ given by Eq. (5) for the same values of K are shown by the dashed lines. The behaviour of the curves $e_p(E)$ obtained when solving problem (1), (2) and determined by (5), is the same. When E decreases from one to zero, which corresponds to a change of x from zero to infinity, the value of $|e_p|$ first increases, and then decreases to zero when $E = 0$, $x = \infty$.

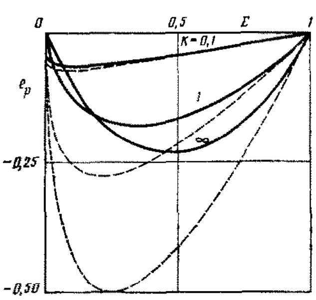

Fig.1

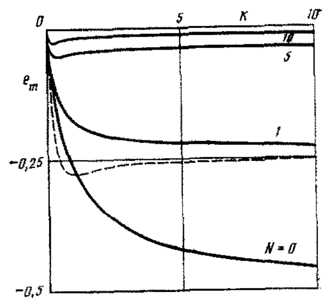

Fig.2

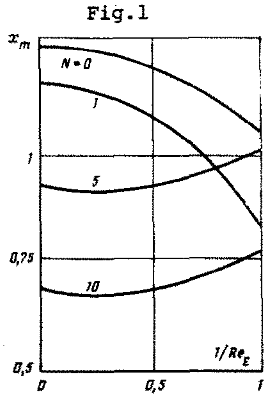

Fig.3

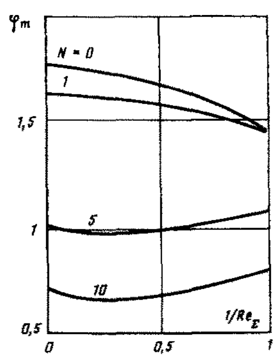

Fig.4

For low values of the dimensionless reaction rate constant K, Eq.(5) yields a better approximation to the relations $e_p(E)$ obtained by numerical methods, than in the case of large K.

Fig.2 shows the dependence of the minimum charge of the particles within the stream $e_m = \min e_p$ on the quantity K for various values of the parameter N (the quantity e_m corresponds to $\max|e_p|$, since $e_p < 0$ when $E^c > 0$). For $N \lesssim 1$, the relation $e_m(K)$ is monotonic and the absolute value of the minimum charge increases with as K increases. For sufficiently large values of N the relation $e_m(K)$ has a minimum, but we have, in all cases, the passage to a finite limit $e_m(\infty)$ as K increases. The dependence $e_m(K)$ obtained by determining the extremum of the function (5) is shown in Fig.2 by the dashed line. The relation is also non-monotonic and has a minimum. When the dimensionless reaction rate constant K tends to zero, the maximum value of the absolute magnitude of the charge on the particles $|e_m|$ obtained in the flow, also always tends to zero.

Figs.3 and 4 show the dependence of the distance x_m from the plane at which the charge on the particles reaches the maximum value in modulus, and of the dimensionless potential difference $\varphi_m = \varphi 8\pi e b n^\circ/(uE)$ on the parameter $1/\mathrm{Re}_E$ for various values of N, where $K=\infty$. We see that the quantity x_m depends weakly on the parameters N, Re_E and $x_m \sim 1$, and the presence of the aerosol particles has a considerable effect on the relation $\varphi_m(1/\mathrm{Re}_E)$ only when $N > 1$ and $\mathrm{Re}_E < 2$.

REFERENCES

1. SEDOV L.I., Mechanics of a Continuous Medium, Moscow, Nauka, 1976.
2. VATAZHIN A.B., GRABOVSKII V.I., LIKHTER V.A. and SHUL'GIN V.I., Electrogasdynamic Flows. Moscow, Nauka, 1983.
3. CHALMERS J.A., Atmospheric Electricity. Oxford, Pergamon Press, 1967.
4. ADACHI M., KOUSAKA Y. and OKUYAMA K., Unipolar and bipolar diffusion charging of ultrafine aerosol particles, J. Aerosol Sci. 16, 2, 1985.
5. SEDOV G.L. and CHERNYI L.T., Equations of electrohydrodynamics of weakly ionized aerosols with diffusive charging of the particles of the disperse phase. Izv. Akad. Nauk SSSR, MZhG, 1, 1986.
6. SMIRNOV B.M., Introduction to Plasma Physics. Moscow, Nauka, 1982.
7. COLE J.D., Perturbation Methods in Applied Mathematics. Waltham, Mass. Blaisdell Pub. Co. 1968.

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THE PROBLEM OF THE FILLING OF A LIMITED VOLUME BY A VISCOUS HEAT-CONDUCTING GAS*

S.YA. BELOV

A system of differential equations, the solution of which describes the one-dimensional motion of a viscous heat-conducting ideal polytropic gas is investigated /1, 2/. It is proved that the problem of the filling of a limited volume by a gas is uniquely solvable. An existence theorem is established by the method of extending the solution that is local in time using global a priori estimates. A method of obtaining these estimates was described in /2/ for the equations of a viscous gas described in Lagrangian variables. The presence of penetrable walls means that the boundary conditions are non-uniform, and in mass Lagrangian variables the initial-boundary value problem is formulated in a region with curvilinear boundaries. This requires the development of a technique for proving the estimates. The correctness in time as a whole of the problem of the filling of a volume by a viscous gas has only been investigated previously for the more simple models, and for the system of equations of a heat-conducting gas in the case when the thermal conductivity depends in a special way on the temperature /3, 4/. Other formulations of the problem of the flows of a viscous gas in regions with penetrable boundaries were studied in /3-6/.

1. Formulation of the problem and fundamental results.

The one-dimensional motion of a viscous ideal polytropic gas in mass Lagrangian coordinates is described by the following system of equations /1, 2/:

$$\frac{\partial u}{\partial t} = \mu \frac{\partial}{\partial x}\left(\rho \frac{\partial u}{\partial x}\right) - \frac{\partial p}{\partial x}, \quad \frac{\partial \rho}{\partial t} + \rho^2 \frac{\partial u}{\partial x} = 0 \tag{1.1}$$

$$c_v \frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(\varkappa\rho \frac{\partial \theta}{\partial x}\right) + \mu\rho\left(\frac{\partial u}{\partial x}\right)^2 - p\frac{\partial u}{\partial x}, \quad p = R\rho\theta$$

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